ESTABLISHED 1922

NEW YORK INSTITUTE OF FINANCE

Corporate Finance & Valuation Methods
Professional Certificate

Fundamental of Financial Mathematics and Capital Budgeting

Program Overview

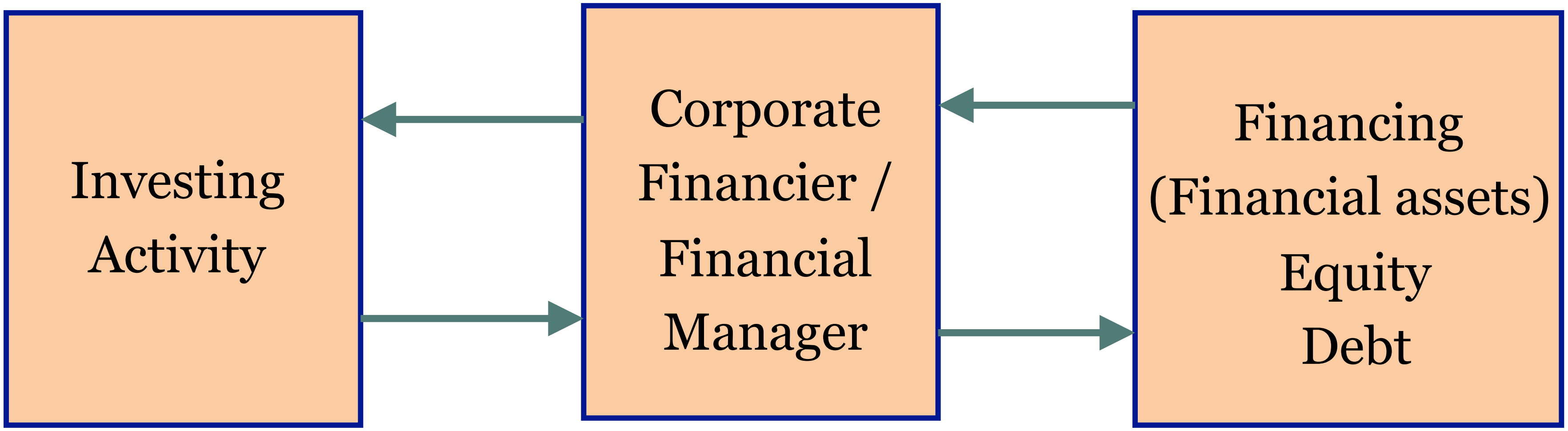
What is Corporate Finance?

- Allocation of financial resources to investment
  - Acquisitions or dispositions of real assets
  - Directly – property, plant or equipment
  - Indirectly – mergers, acquisitions, spin-offs, strategic partnerships, joint ventures, etc.

- Method (and cost) of financing the investments / purchases

- Decisions to invest, acquire, expand, dispose, spin-off, etc. should be driven by goal of maximizing cumulative real returns

Corporate Finance Decisions

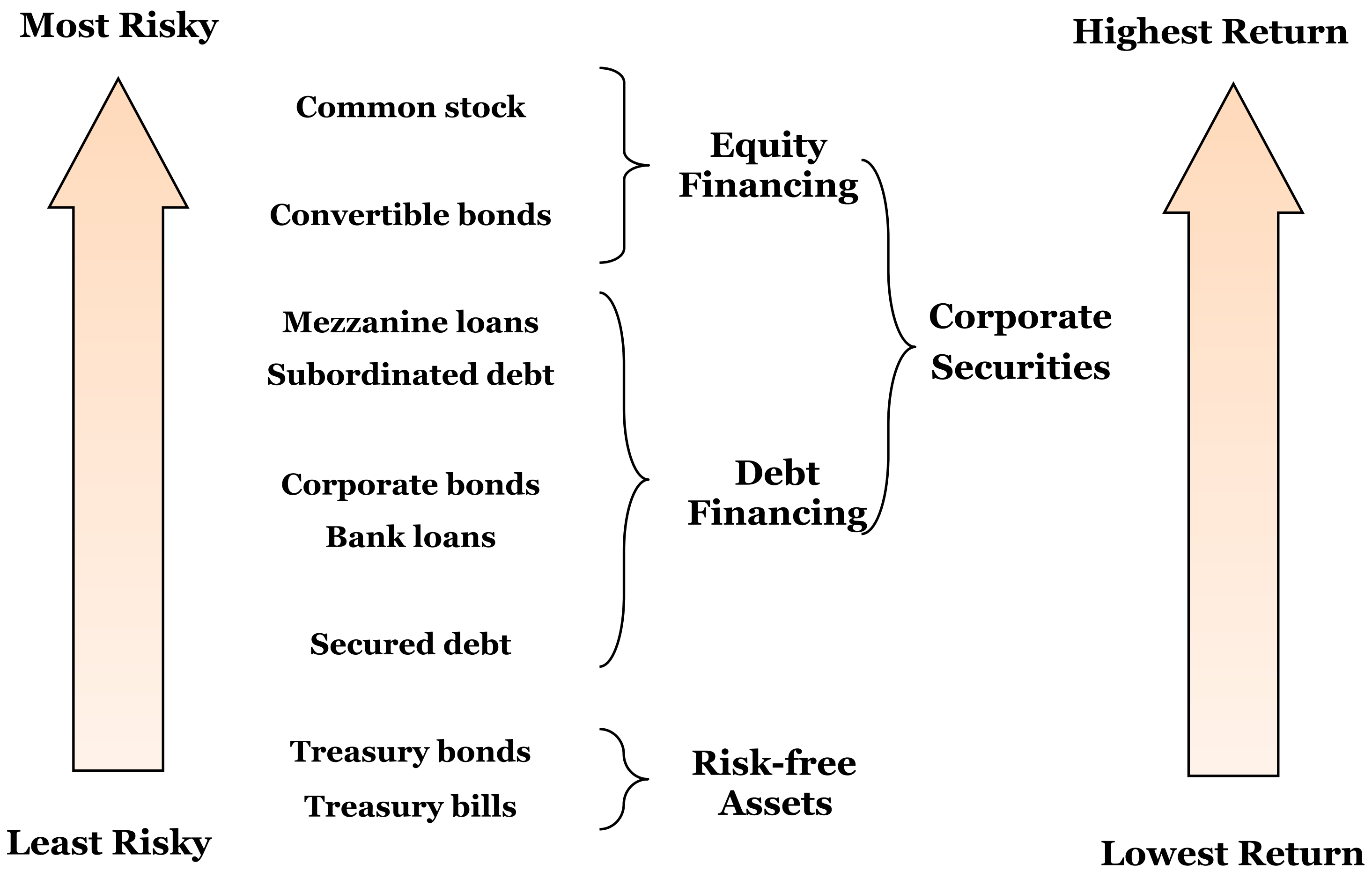


Investing Activity

Corporate Financier / Financial Manager

Financing (Financial assets)
Equity
Debt

Fundamental Principle – Risk vs. Return



Most Risky

Highest Return

Common stock

Convertible bonds

Equity Financing

Mezzanine loans

Subordinated debt

Corporate bonds

Bank loans

Secured debt

Debt Financing

Corporate Securities

Treasury bonds

Treasury bills

Risk-free Assets

Least Risky

Lowest Return

Corporate Finance and Valuation Methods – Program Overview

Day 1

Fundamentals of Financial Mathematics and Capital Budgeting

- Determinants of Value
- Foundations of Financial Mathematics
- Capital Budgeting/Investment Project Selection Methodologies

Day 2

Cost of Capital

- Factors Influencing the Cost of Capital
- Cost of Debt Capital
- Cost of Convertible Debt
- Cost of Equity
- Implications of Cost for Corporate Funding Decisions

Day 3

Discounted Cash Flow and other Valuation Methodologies

- Overview of Discounted Cash Flow Methodologies
- Elements of Cash Flow Projections
- Discounted Cash Flow Analyses
- Alternate Valuation Methodologies
- Relative Valuation Techniques

Day 4

Option Pricing and Applications in Capital Budgeting and Corporate Finance

- Introduction to Option Contracts and Option Pricing
- Real Options
- Integrating Real Options with Net Present Value Analysis
- Corporate Securities Deconstructed as Options

Corporate Finance and Valuation Methods – Program Overview

Day 5

Corporate Funding Strategies

- Financing Privately Held Companies
- Raising Capital in Public Markets
- Other Transactions
- Comprehensive Case Exercise

Day 1

Fundamentals of Financial Mathematics and Capital Budgeting

Day One Learning Objectives

By the completion of this program, you should be able to:

- Differentiate between simple and compound interest
- Solve simple and compound interest problems
- Solve problems using time value of money concepts
- Solve statistical problems
- Differentiate between nominal and effective interest rates
- Calculate the present and future value of an annuity
- Identify the relevant costs in a project
- Calculate the NPV and IRR of a project
- Apply NPV and IRR to determine whether a project should be undertaken
- Explain why MIRR is an improvement over IRR

Day 1 - Fundamentals of Financial Mathematics and Capital Budgeting

1

Determinants of Value

- Elements of Valuation Models
- Interest Rates – an Overview of Uses and Interpretations

2

Foundations of Financial Mathematics

- Future Value of Money in Hand Today (a Lump Sum)
- Future Value of an Annuity
- Present Value of a Single Future Cash Flow (Lump Sum)
- Present Value of an Annuity
- Other Issues: Perpetuities and Probability Weighting

3

Capital Budgeting / Investment Project Selection Methodologies

- Typical Inputs for Capital Budgeting Methods
- Payback
- Discounted Payback
- Internal Rate of Return
- Modified Internal Rate of Return
- Net Present Value

Module 1

Determinants of Value

Valuing Asset – Real and Financial

- Determinants of value – cash flows and risk
- Cash flows
    - Amount and timing
    - Projected operating cash flows
    - Terminal values
        - Disposal values (sale prices/salvage values) or costs
        - Perpetuities – cash flows beyond explicit projection horizon
- Adjusting for risk
    - Future "states" (e.g. best case, worst case, etc.)
    - Discount rates
        - Risk-free rates - time value adjustments
        - Risk premium – reflection of relative risk of project/asset

Valuing Asset – Real and Financial

- Asset valuation methodologies
  - Discounting future cash flows
    - Internal rate of return (IRR)
    - Dividend discount models
    - Net present value/discounted cash flow analysis
  - Elements
    - Future cash flow
    - Discount rate
    - Current cost/required investment outlay
    - Valuation ratios

Discounted Cash Flow Valuation

$$\text{Price}(\text{PV}) = \frac{CF_1}{(1+i)^1} + \frac{CF_2}{(1+i)^2} + \frac{CF_3}{(1+i)^3} + \cdots + \frac{CF_N}{(1+i)^N}$$

CF_J = Cash Flow at time J
Exponent/Subscript = Timing of cash flow – in periods
i = Periodic discount (interest) rate
N = Number of periods over life of investment

Conceptually all investments, real or financial can be valued using the same approach used to price bonds, Value being the present value of the future cash flows

Valuing Asset – Real and Financial

- Valuation ratios
  - Types of valuation ratios
    - Price/earnings (P/E) ratio – earnings multiples
    - Price to book (value) ratio – book multiples
    - Price to cash flow – cash flow multiple
  - Missing elements
    - Future cash flow projections
    - Risk adjustment
  - How the (seemingly) missing elements are captured
    - Reflected in ratio
    - Benchmarks for assessing relative value - comparables

Valuation Ratios

Using P/E ratios to calculate target price and project returns

$$P / E = \frac{\text{Price}}{\text{eps}}$$

⇩

Target price = eps × P/E

That is, the target price (estimated value) at the end of period equals the forecasted earning per share over the subsequent period time the projected forward P/E as of end of period.

Interest Rates

- What is an interest rate?
  - Uses of interest rates
    - Interest payment determination
    - Valuing (and pricing) future cash flows
    - Expression of current values (quotes)
    - Time value of money calculations
  - Definitions
    - Economists' interpretation
    - Wall Street perspective

Interest Rates

Economists	Financial Markets
Compensation for deferring consumption	Opportunity cost of capital
Factor that connects the present to the future	Time value of money

Interest Rates

- What’s an interest rate?
  - Price of money? – frequently used expression, but be careful of the interpretation of that phrasing
  - Price of credit
  - Rental fee/income for the temporary use of someone else’s money

- Risk-free rates and their uses
  - Default risk-free
  - Discounting future cash flow (plus a risk premium if apropos)
  - Nominal return earned with certainty
  - Selecting the appropriate risk-free rate

Interest Rate Conventions

- Simple interest rates
  - Interest on original principal only
  - No interest on interest income (compounding)

- Compound interest rates
  - Impact of reinvesting cash flows
    - Reinvestment income
    - Interest on interest income
    - Compound interest
  - Adjusts for use value or opportunity cost of interest over time
    - Used even in the absence of periodic cash flow (e.g. zero coupon bonds)

Module 2

Foundations of Financial Mathematics: Time Value of Money

Financial Mathematics

Simple Interest versus Rate of Return

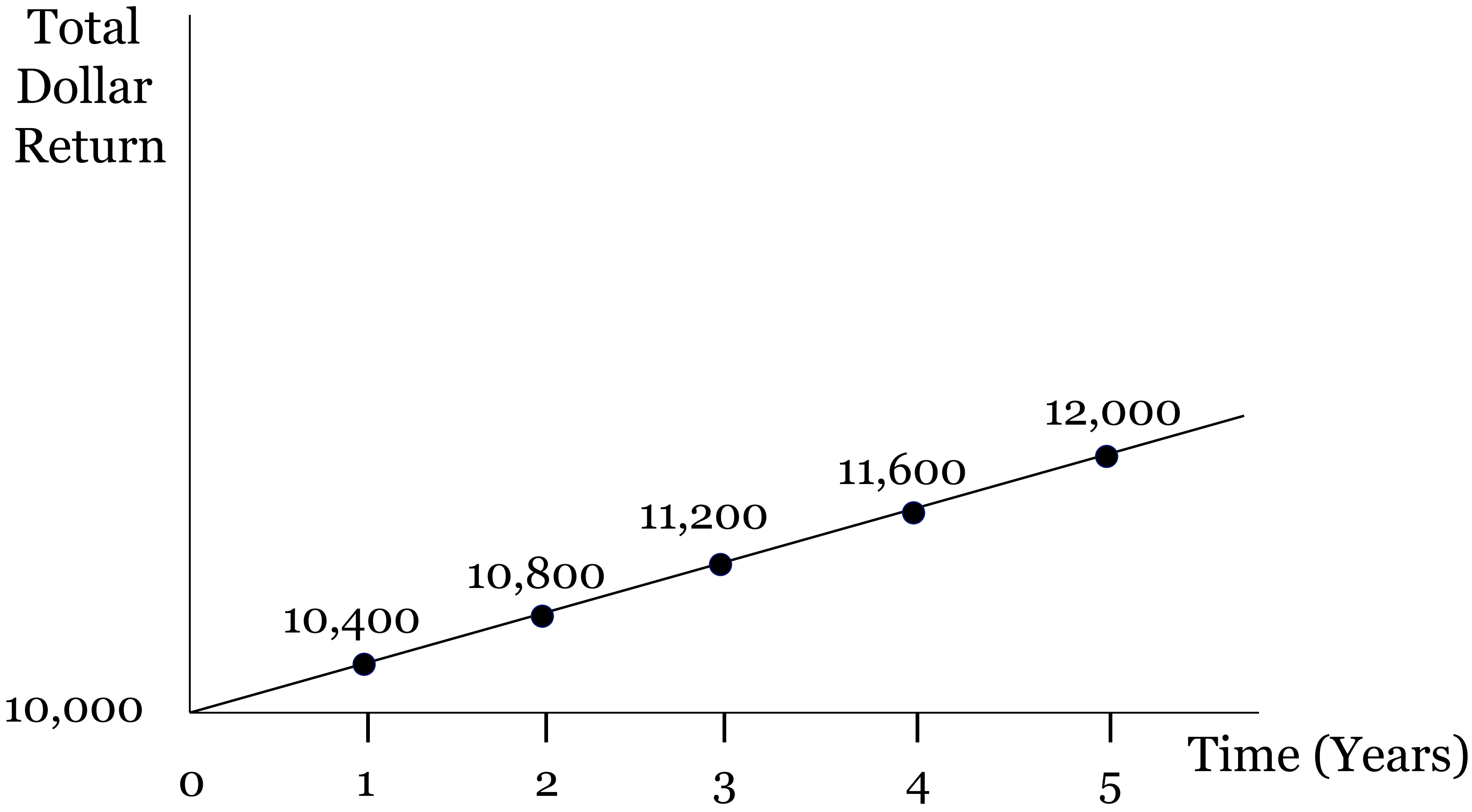
$10,000 invested for 5 years paying 4% simple interest (all interest payable at maturity)

Total Dollar Return
= Principal + Interest (Interest = principal × interest rate × number of periods)
= 10,000 + (10,000 × .04 × 5)
= 10,000 + 2,000
= 12,000

$$\text{Rate of Return} = \left(\frac{\text{Ending value}}{\text{Beginning value}}\right)^{1/N} - 1$$

$$= \left(\frac{12,000}{10,000}\right)^{1/5} - 1 = \left(1.20\right)^{.2} - 1 = 3.71\%$$

Simple Interest



Financial Mathematics

- Time value of money
  - Comparing cash flows of different terms
    - Problems if no adjustment for timing difference
    - Basis for making comparison
  - Present value: the current worth of money payable or receivable in the future
  - Future value: the value at some future date of money in hand today or payable receivable at a different future date

Financial Mathematics

- Time value of money
  - Significant issues
    - Timing of cash flow(s)
    - Interest rate/discount rate
    - Uncertainty/risk
  - Interest rate considerations
    - Compounding assumption
    - Use of a constant rate

Time Value of Money

The rate at which current value (cash/money) can be exchanged for cash/money in the future

More generally, the rate at which value can be exchanged over time

PV = Present value
FV = Future value
i = Rate of interest
N = Number of periods until horizon

Compounding: Finding the future value of money payable/receivable today

$$FV = PV(1+i)^N$$

Discounting: Finding the present value of money payable/receivable at some point in the future

$$PV = \frac{FV}{(1+i)^N}$$

Financial Mathematics

- Future value
  - Single cash flow
  - Multiple cash flows
  - Assumptions
  - Applications
    - Future value of two bonds
    - Relate future value comparison to present value comparison

Compound Interest/Future Value

Capturing the impact of compounding

Each period FDA = P + I (I = P × i × i)

$FDA_1 = P + Pi$

$$\begin{aligned} FDA_2 &= FDA_1 + FDA_1 \times i \\ &= P + Pi + (P + Pi)i \\ &= P + Pi + Pi + Pi^2 \\ &= P + 2Pi + Pi^2 \\ &= P(1+i)^2 \end{aligned}$$

Two period

$FDA_N = P(1+i)^N$ N period

Financial Mathematics

Impact of Compounding

Given $10,000 of principal invested for 5 years at a 4% rate of interest: compare impact of annual to semiannual to monthly compounding.

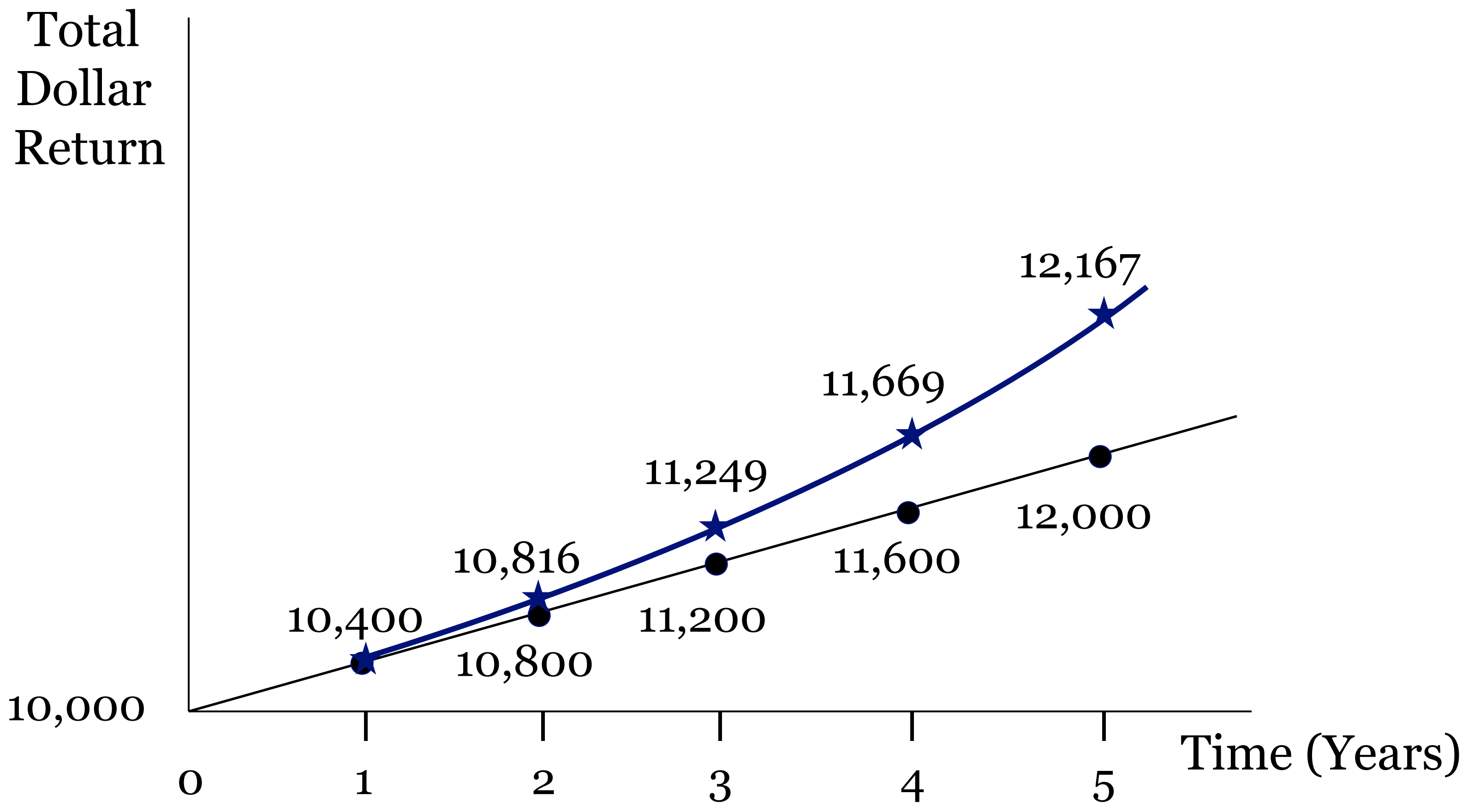
(FV) Future Value = $P(1+i)^N$

$FV_A = 10{,}000(1.04)^5$
$= 10{,}000(1.216652902)$
$= 12{,}166.53$

$FV_{SA} = 10{,}000(1.02)^{10}$
$= 10{,}000(1.218994420)$
$= 12{,}189.94$

$FV_M = 10{,}000(1.00333\ldots)^{60}$
$= 10{,}000(1.22099657)$
$=12{,}209.97$

Simple versus Compound Interests



Financial Mathematics

Impact of Compounding

Given $10,000 of principal invested for 5 years at a 4% rate of interest: compare impact of annual to semiannual to monthly compounding.

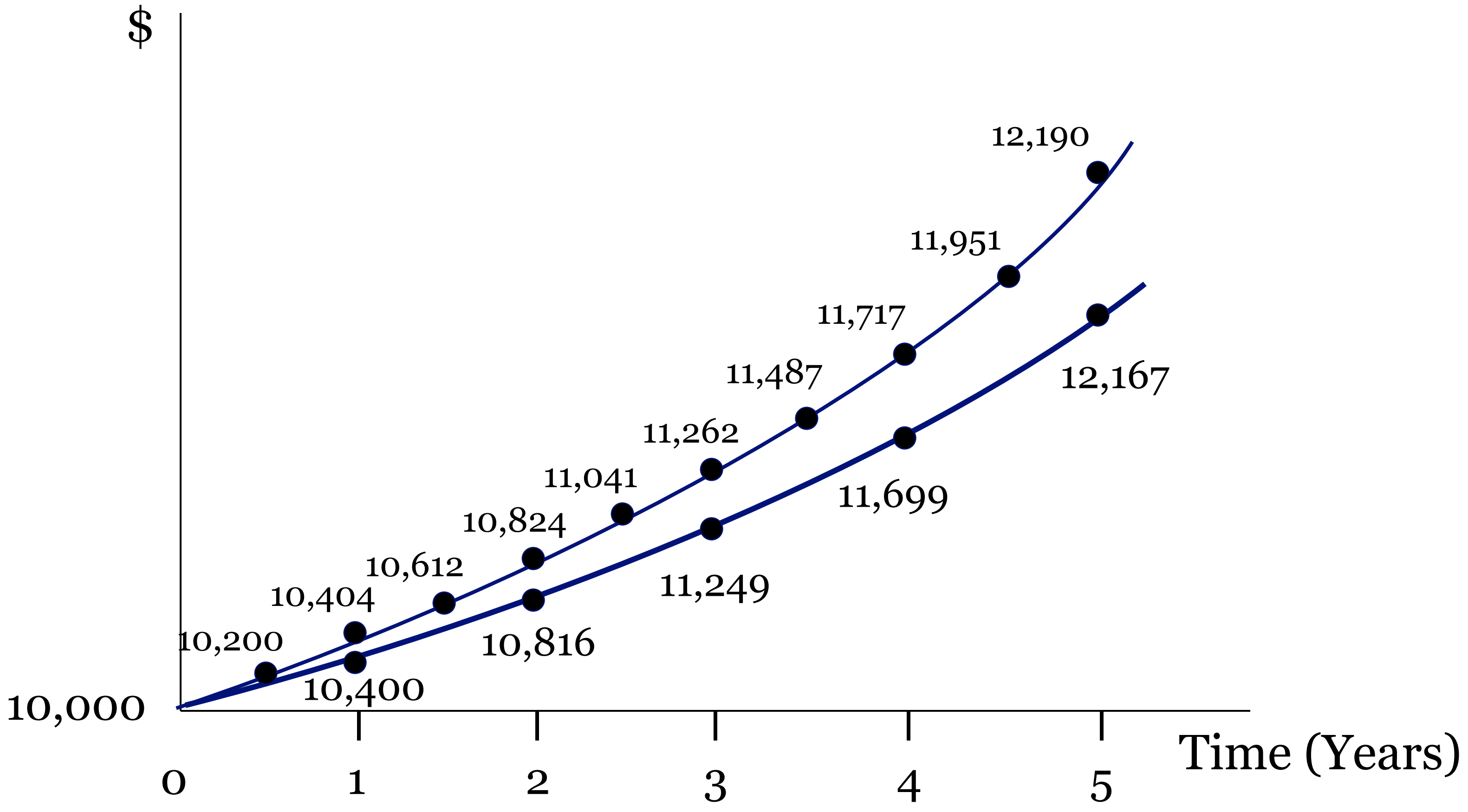
$P(1+i)^N$ = Total Dollar Return
= Principal + Contract interest + Compound interest

Annual
$10{,}000(1.04)^5 = 12{,}166.53 = 10{,}000 + 2{,}000 + 166.53$

Semiannual
$10{,}000(1.02)^{10} = 12{,}189.94 = 10{,}000 + 2{,}000 + 189.94$

Monthly
$10{,}000(1.00333\ldots)^{60} = 12{,}209.97 = 10{,}000 + 2{,}000 + 209.97$

Future Value



Future Value – Exercise

What is the future value of $50,000, 25 years from now, assuming semi-annual compounding and an annual interest rate of 6%?

$$FV = PV \times (1 + i)^N$$

Future Value – Solution

What is the future value of \$50,000, 25 years from now, assuming semiannual compounding and an annual interest rate of 6%?

N = # of years × compounding periods per year

$= 25 \times 2 = 50$

i = annual rate ÷ compounding periods per year

$FV = PV \times (1 + i)^{N}$

$= 50{,}000 \times (1 + .06/2)^{(25 \times 2)}$

$= 50{,}000 \times 4.3839$

$= \$219{,}195.30$

Future Value – Solution (Excel)

	A	B	C	D	E
1					
2					
3		6%	rate		
4		50	nper		
5		0	pmt		
6		-50,000	pv		
7		=FV(B3/2,B4,B5,B6)	$219,195.30		

Financial Mathematics

Effective Annual Rate (or Yield)

Comparing rates based on differing compounding frequencies

Annual = 4.04% Semiannual = 4.00% Monthly = 3.96%

$$\text{Effective Annual Rate} = \left(1 + \frac{\text{Quoted rate}}{\text{Periods per year}}\right)^{\text{PPY}} - 1$$

$$\text{Annual} = \left(1 + \frac{.0404}{1}\right)^{1} - 1 = 1.0404^{1} - 1 = 4.04\%$$

$$\text{Semiannual} = \left(1 + \frac{.04}{2}\right)^{2} - 1 = 1.02^{2} - 1 = 4.04\%$$

$$\text{Monthly} = \left(1 + \frac{.0396}{12}\right)^{12} - 1 = 1.0033^{12} - 1 = 4.0327\%$$

Financial Mathematics

- Comparing different compounding conventions
- Continuous versus periodic discounting

Financial Mathematics

Continuous Compounding

$$\text{Future Value} = P\,e^{rt}$$

$$= 10{,}000e^{(.04)(5)}$$

$$= 10{,}000e^{.2}$$

$$= 10{,}000\ (1.221402758)$$

$$= 12{,}214.03$$

Exercise One

Individual exercise on simple interest, compound interest and time value of money

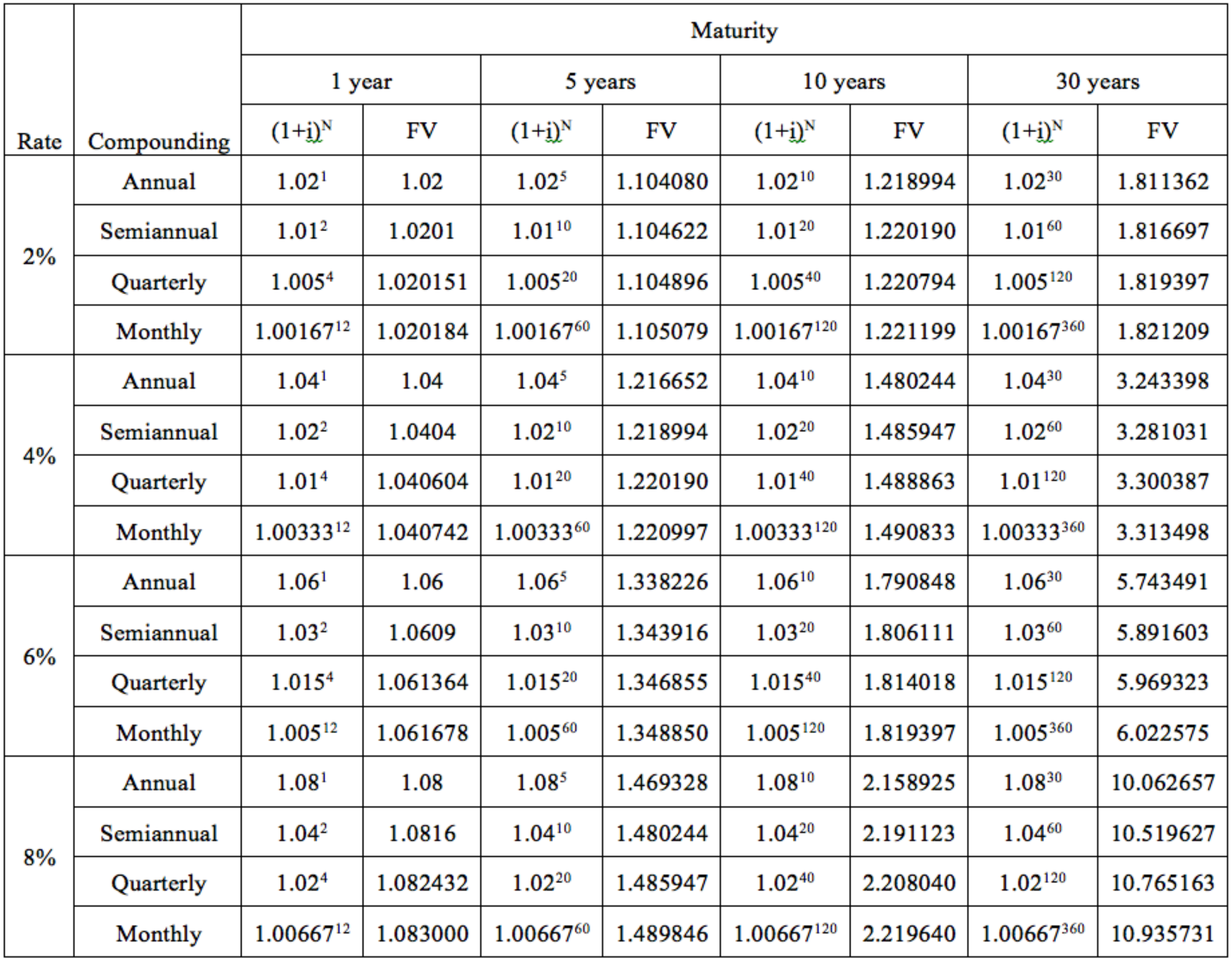
Future Value of Money Today

Future value of \$1 assuming a 4% interest rate

Compounding Frequency	Maturity			
	1 year	5 years	10 years	30 years
Annual	$(1.04)^{1}$ 1.04	$(1.04)^{5}$ 1.216653	$(1.04)^{10}$ 1.480244	$(1.04)^{30}$ 3.243398
Semiannual	$(1.02)^{2}$ 1.0404	$(1.02)^{10}$ 1.218994	$(1.02)^{20}$ 1.485947	$(1.02)^{60}$ 3.281031
Quarterly	$(1.01)^{4}$ 1.040604	$(1.01)^{20}$ 1.220190	$(1.01)^{40}$ 1.488864	$(1.01)^{120}$ 3.300387
Monthly	$(1.00333)^{12}$ 1.040742	$(1.00333)^{60}$ 1.220997	$(1.00333)^{120}$ 1.490833	$(1.00333)^{360}$ 3.313498
Daily	$(1.00011)^{365}$ 1.040808	$(1.00011)^{1825}$ 1.221389	$(1.00011)^{3650}$ 1.491792	$(1.00011)^{10,950}$ 3.319897
Continuous	$e^{.04(1)}$ 1.040811	$e^{.04(5)}$ 1.221403	$e^{.04(10)}$ 1.491825	$e^{.04(30)}$ 3.320117

Future Value of Money Today

Future value of $1 invested today for different investment horizons at different compounding frequencies



Rate	Compounding	Maturity							
		1 year		5 years		10 years		30 years	
		$(1+i)^N$	FV	$(1+i)^N$	FV	$(1+i)^N$	FV	$(1+i)^N$	FV
2%	Annual	1.02^{1}	1.02	1.02^{5}	1.104080	1.02^{10}	1.218994	1.02^{30}	1.811362
	Semiannual	1.01^{2}	1.0201	1.01^{10}	1.104622	1.01^{20}	1.220190	1.01^{60}	1.816697
	Quarterly	1.005^{4}	1.020151	1.005^{20}	1.104896	1.005^{40}	1.220794	1.005^{120}	1.819397
	Monthly	1.00167^{12}	1.020184	1.00167^{60}	1.105079	1.00167^{120}	1.221199	1.00167^{360}	1.821209
4%	Annual	1.04^{1}	1.04	1.04^{5}	1.216652	1.04^{10}	1.480244	1.04^{30}	3.243398
	Semiannual	1.02^{2}	1.0404	1.02^{10}	1.218994	1.02^{20}	1.485947	1.02^{60}	3.281031
	Quarterly	1.01^{4}	1.040604	1.01^{20}	1.220190	1.01^{40}	1.488863	1.01^{120}	3.300387
	Monthly	1.00333^{12}	1.040742	1.00333^{60}	1.220997	1.00333^{120}	1.490833	1.00333^{360}	3.313498
6%	Annual	1.06^{1}	1.06	1.06^{5}	1.338226	1.06^{10}	1.790848	1.06^{30}	5.743491
	Semiannual	1.03^{2}	1.0609	1.03^{10}	1.343916	1.03^{20}	1.806111	1.03^{60}	5.891603
	Quarterly	1.015^{4}	1.061364	1.015^{20}	1.346855	1.015^{40}	1.814018	1.015^{120}	5.969323
	Monthly	1.005^{12}	1.061678	1.005^{60}	1.348850	1.005^{120}	1.819397	1.005^{360}	6.022575
8%	Annual	1.08^{1}	1.08	1.08^{5}	1.469328	1.08^{10}	2.158925	1.08^{30}	10.062657
	Semiannual	1.04^{2}	1.0816	1.04^{10}	1.480244	1.04^{20}	2.191123	1.04^{60}	10.519627
	Quarterly	1.02^{4}	1.082432	1.02^{20}	1.485947	1.02^{40}	2.208040	1.02^{120}	10.765163
	Monthly	1.00667^{12}	1.083000	1.00667^{60}	1.489846	1.00667^{120}	2.219640	1.00667^{360}	10.935731

Financial Mathematics

- Future value of a stream of cash flows
  - Future value of an annuity (period certain)
    - Annuity payments plus reinvestment income
    - Payment (compounding frequency) and interest rate
  - Future value of a bond
    - Future value of the coupons (future value of an annuity)
    - Future value of the principal (dollar amount of principal at maturity if computed to that time)
  - Meaning and uses of a bond's future value

Future Value of an Annuity

$$FV = PMT_1(1 + i)^N + PMT_2(1 + i)^{N-1} + \ldots + PMT_N(1 + i)^{N-(N-1)}$$

$$FV_{25} = \$25\ (1.03)^6 + \$25\ (1.03)^{6-1} + \ldots + \$25\ (1.03)^{6-5}$$

$$= \quad \$29.85 \quad + \quad \$28.98 \quad + \ \ldots \ + \ \$25.75$$

$$= \ \$166.56$$

$$FV_{45} = \$45\ (1.03)^6 + \$45\ (1.03)^{6-1} + \ldots + \$45\ (1.03)^{6-5}$$

$$= \quad \$53.73 \quad + \quad \$52.17 \quad + \ \ldots \ + \$46.35$$

$$= \ \$299.81$$

Future Value of an Annuity in Arrears

$$FV = C_1(1 + i)^{N-1} + C_2(1 + i)^{N-2} + \ldots + C_N(1 + i)^{N-N}$$

$$FV_{25} = \$25\ (1.03)^{6-1} + \$25\ (1.03)^{6-2} + \ldots + \$25\ (1.03)^{6-6}$$

$$= \quad \$28.98 \quad + \quad \$28.14 \quad + \ldots + \ \$25.00$$

$$= \ \$161.71$$

$$FV_{45} = \$45\ (1.03)^{6-1} + \$45\ (1.03)^{6-2} + \ldots + \$45\ (1.03)^{6-6}$$

$$= \quad \$52.17 \quad + \quad \$50.65 \quad + \ldots + \ \$45.00$$

$$= \ \$291.08$$

FV of an Annuity – Formula

$$FV\,(An) = CF\left[\frac{(1+i)^{N} - 1}{i}\right]$$

Where:

FV (An) = Future value of a stream of annuity payments

CF = Cash flow of annuity payment

i = Periodic discount rate or interest rate

N = Number of periods over which annuity payments are made

FV of an Annuity – Exercise

How much money will you have if you invest $6,000 per year at 10% for 15 years?

Assume:

- Annual compounding
- The first $6,000 invested one year from today

FV of an Annuity – Solution

$$FV\,(An) = CF\left[\frac{(1+i)^{N} - 1}{i}\right]$$

$$FV = 6{,}000 \times [[(1+0.10)^{15} - 1] / 0.10]$$

$$FV = 6{,}000 \times [\,(4.177 - 1) / 0.10\,]$$

$$FV = 6{,}000 \times [\,3.177 / 0.10\,]$$

$$FV = 6{,}000 \times 31.77$$

$$FV = 190{,}635$$

FV of an Annuity – Solution (Excel)

	A	B	C	D
1				
2	10%	rate		
3	15	nper		
4	6000	pmt		
5	0	pv		
6	1	type		
7	=FV(A2,A3,A4,A5,A6)	($190,634.89)		

Bond Future Values

3 years to maturity, semiannual-pay bonds,
trading at 6% yields to maturity
5% and 9% coupon rates

$$FV = C_1(1+i)^{N-1} + C_2(1+i)^{N-2} + \ldots + C_N(1+i)^{N-N} + \text{Principal}(1+i)^{N-N}$$

$FV_{5\%}$

$$= 25(1.03)^5 + 25(1.03)^4 + 25(1.03)^3 + 25(1.03)^2 + 25(1.03)^1 + 25(1.03)^0 + 1{,}000(1.03)^0$$

$$= 1{,}161.71$$

$FV_{9\%}$

$$= 45(1.03)^5 + 45(1.03)^4 + 45(1.03)^3 + 45(1.03)^2 + 45(1.03)^1 + 45(1.03)^0 + 1{,}000(1.03)^0$$

$$= 1{,}291.08$$

Financial Mathematics

Future Value of a Bond

Bond A		**Bond B**
$1,000 par		$1,000 par
3 years		3 years
5% coupon		9% coupon
6% YTM		6% YTM
FV 1,161.71		FV 1,291.08
Price 972.91		Price 1,081.25
$\frac{1,161.71}{972.91}$	FV/$1 invested	$\frac{1,291.08}{1,081.26}$
=		=
1.19		1.19

Financial Mathematics

- Present value
  - Sample calculations
    - Single cash flow investments – zero coupon bonds
    - Multiple cash flows – coupons plus principal
  - Fixed income/derivative pricing applications
  - Basis for discounted cash flow valuation

Present Value

Present value (PV) formula can be derived from the future value (FV) formula

$$FV = P(1 + i)^N$$

$$\frac{FV}{(1+i)^N} = \frac{P\cancel{(1+i)^N}}{\cancel{(1+i)^N}}$$

$$\frac{FV}{(1+i)^N} = \text{Price or PV}$$

Present Value of a Lump Sum

FV = $10,000 N = 1 i = 4% (annual)

$$\begin{aligned} PV &= \frac{FV}{(1+i)^N} \\ &= \frac{\$10,000}{1.04^1} \\ &= \$9,615.38 \end{aligned}$$

Obtaining Present Values Using a Present Value Factor or Discount Factor

$$
\begin{aligned}
\text{PV} &= \text{FV} \times \frac{1}{(1+\text{i})^{\text{N}}} \\
&= \$10,000 \times \frac{1}{1.04^{1}} \\
&= \$10,000 \times .961538 \\
&= \$9,615.38
\end{aligned}
$$

Present Value Factor (PVF) or Discount Factor (DF)

$$
\begin{aligned}
\text{PVF or DF} &= \frac{1}{(1+\text{i})^{\text{N}}} \\
&= \frac{1}{1.04} = .961538
\end{aligned}
$$

Present Value of a Lump Sum

FV = \$10,000 N = 5 i = 4% (annual)

$$PV = \frac{FV}{(1+i)^N}$$

$$= \frac{\$10,000}{1.04^5} = \frac{\$10,000}{1.216653} = \$8,219.27$$

Or

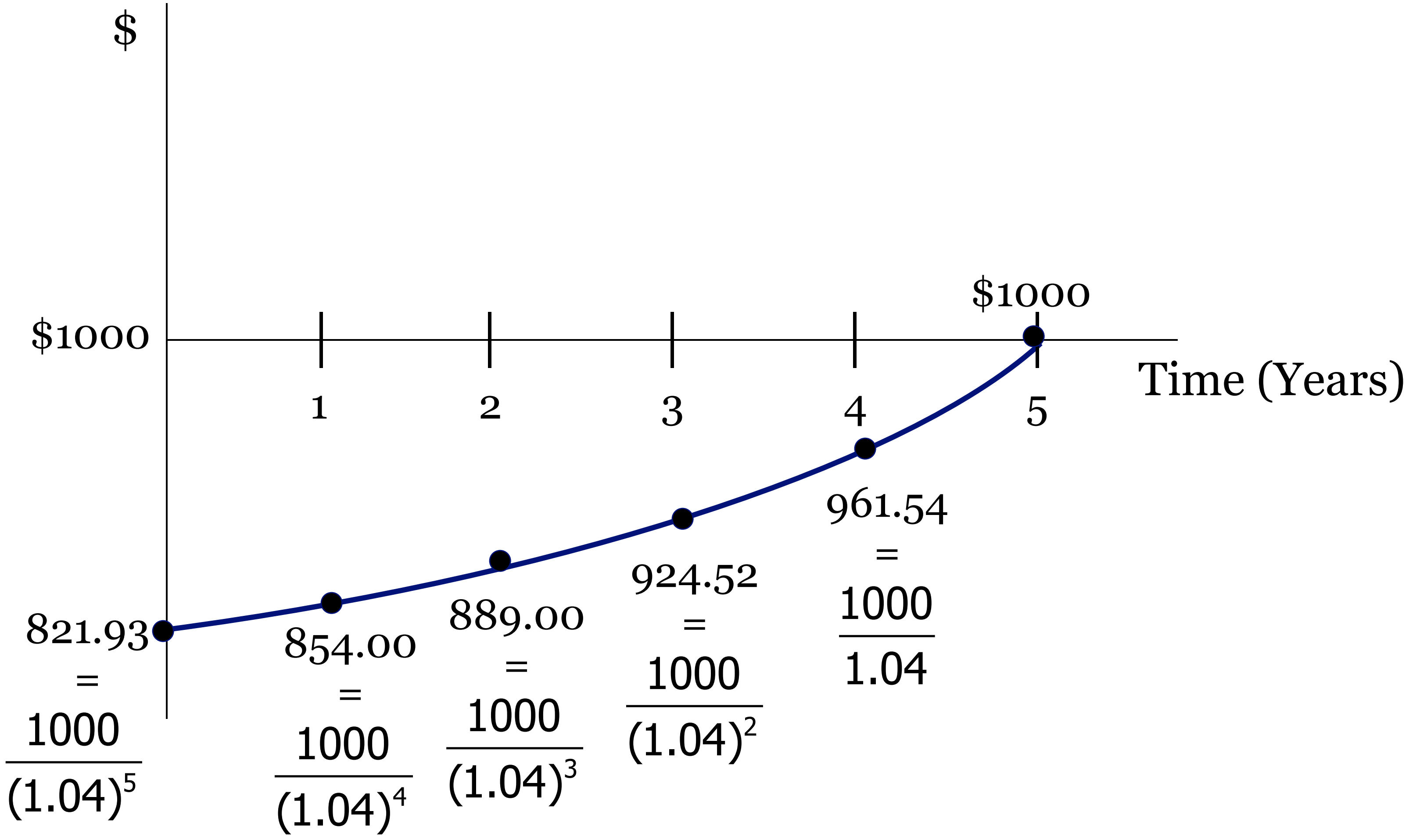
$$PV = FV \times PVF \text{ (or DF)}$$

$$= \$10,000 \times \frac{1}{1.04^5}$$

$$= \$10,000 \times .821927$$

$$= \$8,219.27$$

Present Value



Present Value – Exercise

What is the present value of $200,000 to be received 10 years from now if interest rates are 7.2 % compounded annually?

$$PV = FV / (1 + i)^N$$

Present Value – Solution

What is the present value of $200,000 to be received 10 years from now if interest rates are 7.2 % compounded annually?

$PV = FV / (1 + i)^N$

$PV = 200{,}000 / (1 + .072)^{10}$

$PV = 200{,}000 / 2.004231362$

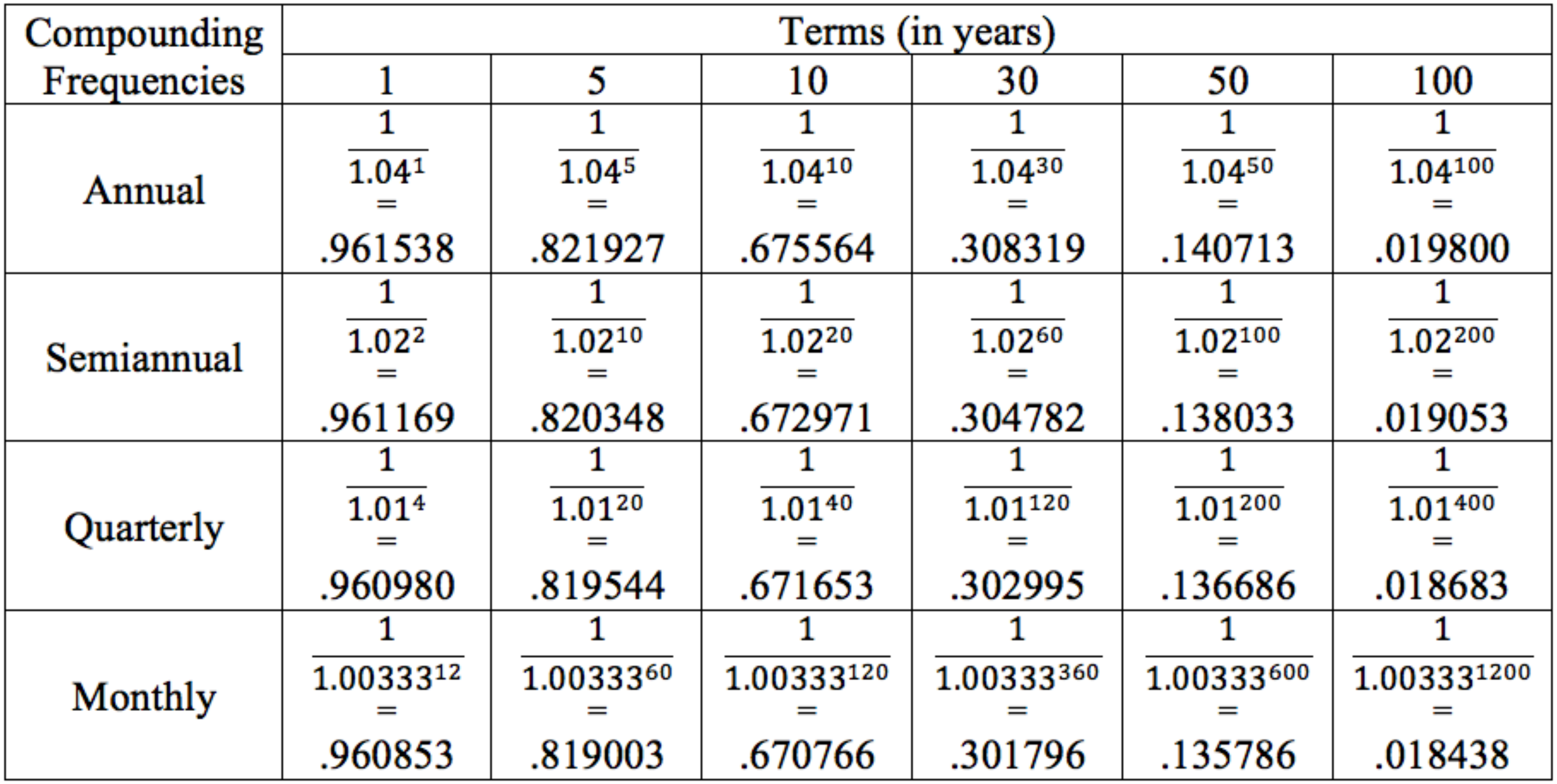
$PV = \$99{,}788.88$

Present Value – Solution (Excel)

	A	B	C	D	E
1	7.2%	rate			
2	10	nper			
3	0	pmt			
4	200,000	fv			
5	1	type			
6	=PV(A1,A2,A3,A4,A5)	($99,788.88)			
7					

Present Values

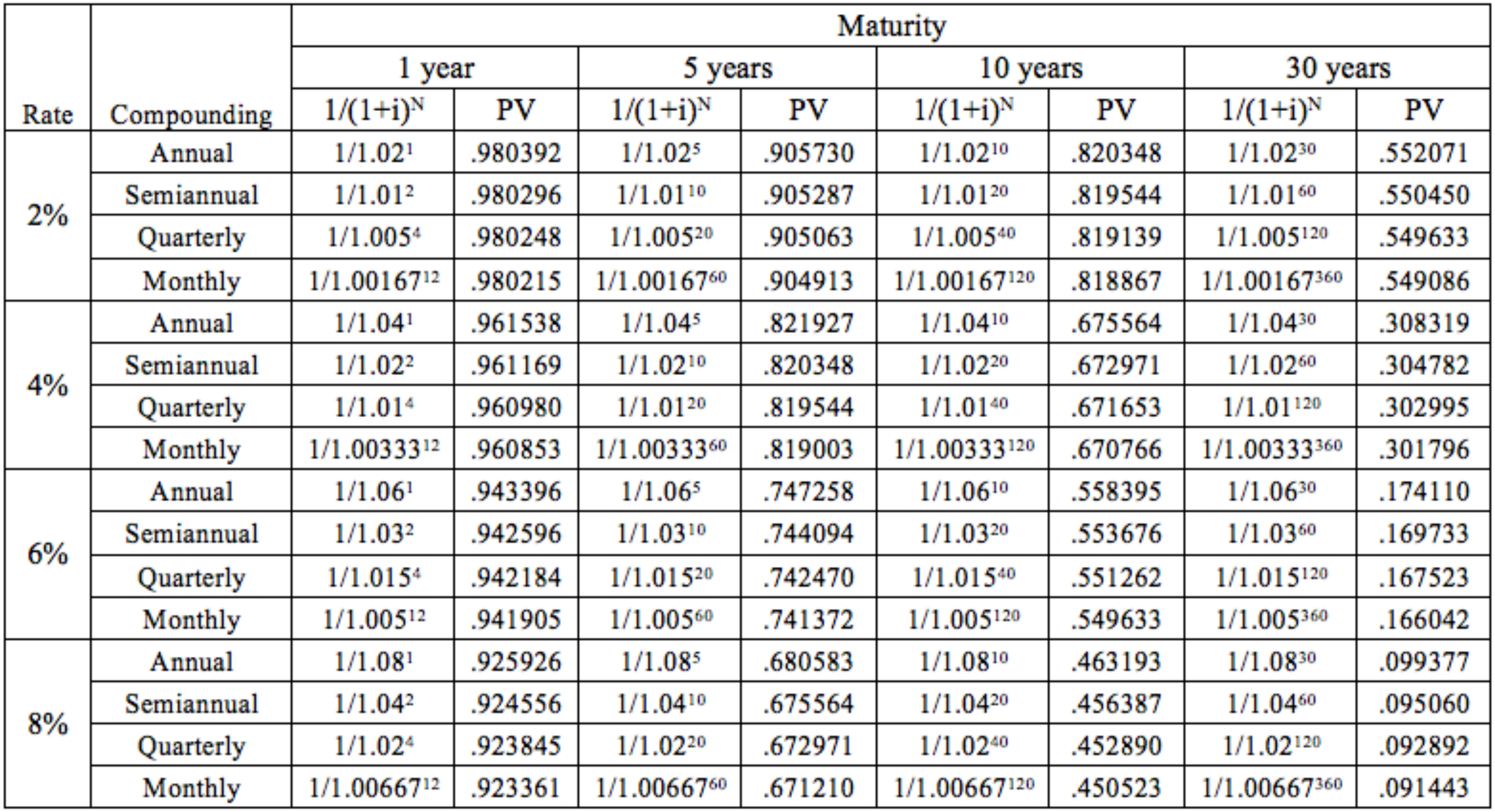
Present value of $1 for various maturities and compounding frequencies based on a 4% annual rate (equivalently present value factors or discount factors)



Compounding Frequencies	Terms (in years)					
	1	5	10	30	50	100
Annual	$\frac{1}{1.04^{1}}$ = .961538	$\frac{1}{1.04^{5}}$ = .821927	$\frac{1}{1.04^{10}}$ = .675564	$\frac{1}{1.04^{30}}$ = .308319	$\frac{1}{1.04^{50}}$ = .140713	$\frac{1}{1.04^{100}}$ = .019800
Semiannual	$\frac{1}{1.02^{2}}$ = .961169	$\frac{1}{1.02^{10}}$ = .820348	$\frac{1}{1.02^{20}}$ = .672971	$\frac{1}{1.02^{60}}$ = .304782	$\frac{1}{1.02^{100}}$ = .138033	$\frac{1}{1.02^{200}}$ = .019053
Quarterly	$\frac{1}{1.01^{4}}$ = .960980	$\frac{1}{1.01^{20}}$ = .819544	$\frac{1}{1.01^{40}}$ = .671653	$\frac{1}{1.01^{120}}$ = .302995	$\frac{1}{1.01^{200}}$ = .136686	$\frac{1}{1.01^{400}}$ = .018683
Monthly	$\frac{1}{1.00333^{12}}$ = .960853	$\frac{1}{1.00333^{60}}$ = .819003	$\frac{1}{1.00333^{120}}$ = .670766	$\frac{1}{1.00333^{360}}$ = .301796	$\frac{1}{1.00333^{600}}$ = .135786	$\frac{1}{1.00333^{1200}}$ = .018438

Present Value

Present value of $1 for different investment horizons
At different compounding frequencies



Rate	Compounding	Maturity							
		1 year		5 years		10 years		30 years	
		$1/(1+i)^N$	PV	$1/(1+i)^N$	PV	$1/(1+i)^N$	PV	$1/(1+i)^N$	PV
2%	Annual	$1/1.02^{1}$	.980392	$1/1.02^{5}$	.905730	$1/1.02^{10}$	.820348	$1/1.02^{30}$	.552071
	Semiannual	$1/1.01^{2}$	.980296	$1/1.01^{10}$	.905287	$1/1.01^{20}$	.819544	$1/1.01^{60}$	.550450
	Quarterly	$1/1.005^{4}$	.980248	$1/1.005^{20}$	.905063	$1/1.005^{40}$	.819139	$1/1.005^{120}$	.549633
	Monthly	$1/1.00167^{12}$	.980215	$1/1.00167^{60}$	.904913	$1/1.00167^{120}$	.818867	$1/1.00167^{360}$	.549086
4%	Annual	$1/1.04^{1}$	.961538	$1/1.04^{5}$	.821927	$1/1.04^{10}$	.675564	$1/1.04^{30}$	.308319
	Semiannual	$1/1.02^{2}$	.961169	$1/1.02^{10}$	.820348	$1/1.02^{20}$	.672971	$1/1.02^{60}$	.304782
	Quarterly	$1/1.01^{4}$	.960980	$1/1.01^{20}$	.819544	$1/1.01^{40}$	.671653	$1/1.01^{120}$	.302995
	Monthly	$1/1.00333^{12}$	.960853	$1/1.00333^{60}$	.819003	$1/1.00333^{120}$	.670766	$1/1.00333^{360}$	.301796
6%	Annual	$1/1.06^{1}$	.943396	$1/1.06^{5}$	.747258	$1/1.06^{10}$	.558395	$1/1.06^{30}$	.174110
	Semiannual	$1/1.03^{2}$	.942596	$1/1.03^{10}$	.744094	$1/1.03^{20}$	.553676	$1/1.03^{60}$	.169733
	Quarterly	$1/1.015^{4}$	.942184	$1/1.015^{20}$	.742470	$1/1.015^{40}$	.551262	$1/1.015^{120}$	.167523
	Monthly	$1/1.005^{12}$	.941905	$1/1.005^{60}$	.741372	$1/1.005^{120}$	.549633	$1/1.005^{360}$	.166042
8%	Annual	$1/1.08^{1}$	.925926	$1/1.08^{5}$	.680583	$1/1.08^{10}$	.463193	$1/1.08^{30}$	.099377
	Semiannual	$1/1.04^{2}$	.924556	$1/1.04^{10}$	.675564	$1/1.04^{20}$	.456387	$1/1.04^{60}$	.095060
	Quarterly	$1/1.02^{4}$	.923845	$1/1.02^{20}$	.672971	$1/1.02^{40}$	.452890	$1/1.02^{120}$	.092892
	Monthly	$1/1.00667^{12}$	.923361	$1/1.00667^{60}$	.671210	$1/1.00667^{120}$	.450523	$1/1.00667^{360}$	.091443

Present Value of an Annuity

5-year annual-pay annuity in arrears
$10,000 annual payment, 4% annual compound rate

$$PV = \frac{FV_1}{(1+i)^1} + \frac{FV_2}{(1+i)^2} + \frac{FV_3}{(1+i)^3} + \frac{FV_4}{(1+i)^4} + \frac{FV_5}{(1+i)^5}$$

$$= \frac{10,000}{1.04^1} + \frac{10,000}{1.04^2} + \frac{10,000}{1.04^3} + \frac{10,000}{1.04^4} + \frac{10,000}{1.04^5}$$

$$= 9,615.38 + 9,245.56 + 8,889.96 + 8,548.04 + 8,219.27$$

$$= 44,518.21$$

PV of an Annuity - Formula

$$PV\,(An) = CF\left[\frac{1}{i} - \left(\frac{1}{i} \times \frac{1}{(1+i)^N}\right)\right]$$

Where:

PV (An) = Preset value of a stream of annuity payments

CF = Cash flow of annuity payment

i = Period discount rate or interest rate

N = Number of periods over which annuity payments are made

Present Value of an Annuity - Exercise

How much would you pay for a 5 year, $100,000 annuity, when the discount rate is 12%?

$$PV\,(An) = CF\left[\frac{1}{i} - \left(\frac{1}{i} \times \frac{1}{(1+i)^{N}}\right)\right]$$

Present Value of an Annuity - Solution

How much would you pay for a 5 year, $100,000 annuity, when the discount rate is 12%?

$$PV\ (An) = CF\left[\frac{1}{i} - \left(\frac{1}{i} \times \frac{1}{\left(1+i\right)^{N}}\right)\right]$$

$= 100{,}000\ [(1/0.12) – [(1/0.12) \times (1/(1.12)^{5})]$

$= 100{,}000\ [8.333 – (8.333 \times 0.567)]$

$= 100{,}000\ \times [8.333 – 4.728]$

$= 100{,}000 \times 3.605$

$= 360{,}478$

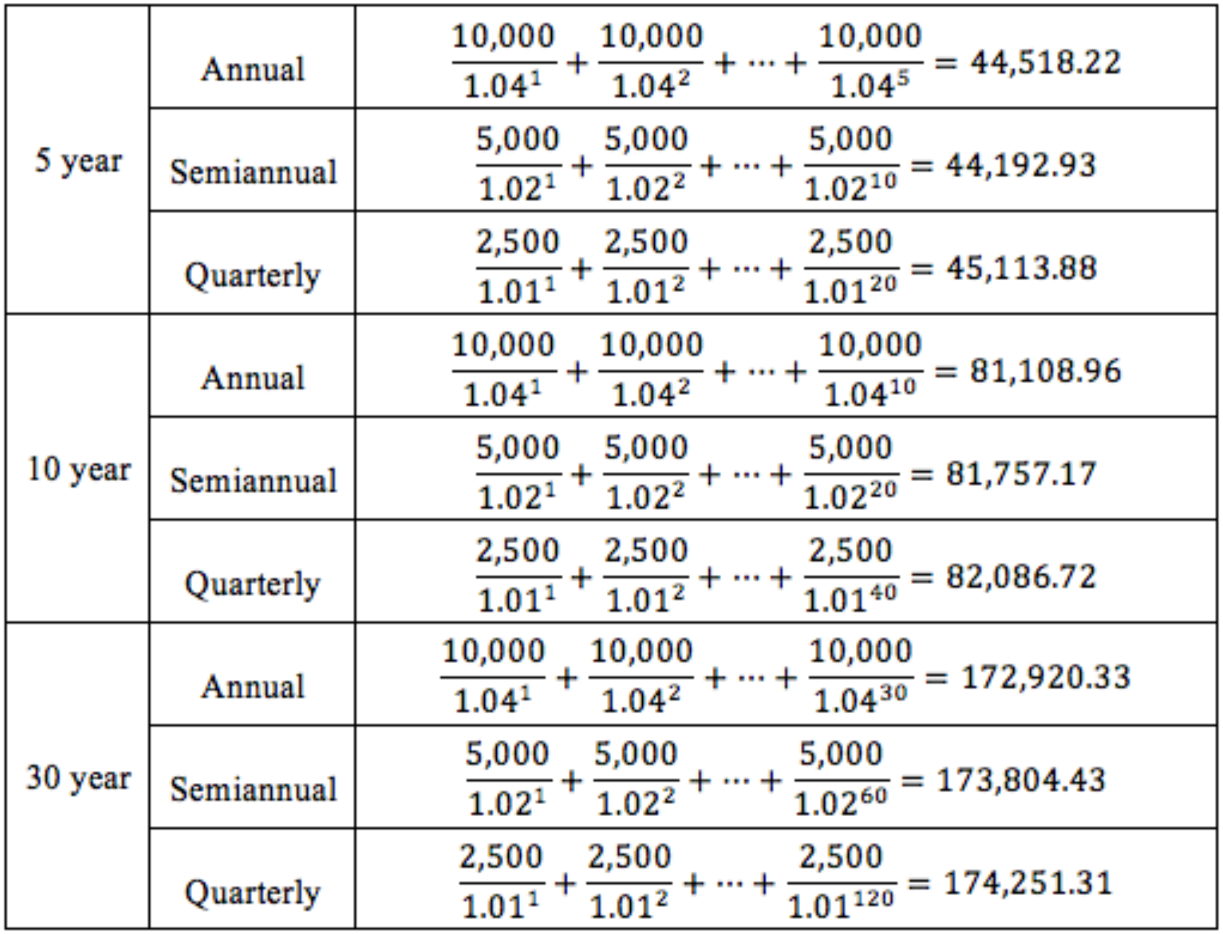
Present Value of an Annuity – Solution (Excel)

	A	B	C	D	E
1					
2	12%	rate			
3	5	nper			
4	100000	pmt			
5	0	fv			
6	0	type			
7	=PV(A2,A3,A4,A5,A6)	($360,477.62)			

Present Value of Annuities

Fixed Annuities: 5-, 10- and 30-year period certain
10,000 per year, annual, semiannual and quarterly-pay, 4% annual rate

$$PV = \frac{FV_1}{(1+i)^1} + \frac{FV_2}{(1+i)^2} + \frac{FV_3}{(1+i)^3} + \frac{FV_4}{(1+i)^4} + \frac{FV_5}{(1+i)^5}$$



5 year	Annual	$\frac{10{,}000}{1.04^1} + \frac{10{,}000}{1.04^2} + \cdots + \frac{10{,}000}{1.04^5} = 44{,}518.22$
	Semiannual	$\frac{5{,}000}{1.02^1} + \frac{5{,}000}{1.02^2} + \cdots + \frac{5{,}000}{1.02^{10}} = 44{,}192.93$
	Quarterly	$\frac{2{,}500}{1.01^1} + \frac{2{,}500}{1.01^2} + \cdots + \frac{2{,}500}{1.01^{20}} = 45{,}113.88$
10 year	Annual	$\frac{10{,}000}{1.04^1} + \frac{10{,}000}{1.04^2} + \cdots + \frac{10{,}000}{1.04^{10}} = 81{,}108.96$
	Semiannual	$\frac{5{,}000}{1.02^1} + \frac{5{,}000}{1.02^2} + \cdots + \frac{5{,}000}{1.02^{20}} = 81{,}757.17$
	Quarterly	$\frac{2{,}500}{1.01^1} + \frac{2{,}500}{1.01^2} + \cdots + \frac{2{,}500}{1.01^{40}} = 82{,}086.72$
30 year	Annual	$\frac{10{,}000}{1.04^1} + \frac{10{,}000}{1.04^2} + \cdots + \frac{10{,}000}{1.04^{30}} = 172{,}920.33$
	Semiannual	$\frac{5{,}000}{1.02^1} + \frac{5{,}000}{1.02^2} + \cdots + \frac{5{,}000}{1.02^{60}} = 173{,}804.43$
	Quarterly	$\frac{2{,}500}{1.01^1} + \frac{2{,}500}{1.01^2} + \cdots + \frac{2{,}500}{1.01^{120}} = 174{,}251.31$

Annuity Payment Determination

Fixed Annuity Period Certain
Periodic Payments

$$CF_j = PV_o \times \frac{(1+i)^N \times i}{(1+i)^N - 1}$$

CF_j = Payment per period
PV_o = Value of annuity at T= 0 (Amount invested)
i = Periodic rate (Annual rate ÷ number of annual payments)
N = Number of payment periods over the life of the mortgage

Annuity Payment – Application

Mortgage Cash Flows
Monthly Payment

$$MP_j = MB_o \times \frac{(1+i)^N \times i}{(1+i)^N - 1}$$

MP_j = Monthly payment for each month
MB_o = Mortgage balance at time = 0 (Amount borrowed)
i = Periodic rate (Annual rate ÷ number of annual payments)
N = Number of payment periods over the life of the mortgage

Annuity Payments

Mortgage Cash Flows
Monthly Payment

$$MP_j = MB_0 \times \frac{(1+i)^N \times i}{(1+i)^N - 1}$$

$$= 400,000 \times \frac{(1.004)^{360} \times .004}{(1.004)^{360} - 1}$$

$$= 2,098.66$$

Assume:

\$400,000 monthly-pay 30-year mortgage with a 4.80% annual rate

$MB_0 = 400,000$

$N = 360 \ (30 \times 12)$

$$i = .004 \left(\frac{.048}{12} \right)$$

Bond Prices and Yields

Pricing a Zero Coupon Bond

Price a 10-year treasury strip trading at a yield to maturity of 3.20%.

$$P = \frac{FV}{(1+i)^N} = \frac{100{,}000}{(1.016)^{20}} = 72{,}799.07$$

What would be the annual rate of return on that treasury strip if held to maturity? 3.20%? Let’s see.

Rate of Return

$$\text{Annualized Rate of Return (ROR)} = \left(\frac{\text{Ending value}}{\text{Beginning value}}\right)^{1/N} - 1$$

$$\frac{\text{Ending value}}{\text{Beginning value}} = 1 + \text{Total return}$$

$$(1 + \text{Total return})^{1/N} = 1 + \text{Annual return}$$

Bond Prices and Yields

$$\text{Annualized Rate of Return} = \left(\frac{FV}{P}\right)^{1/N} - 1$$

$$= \left(\frac{100{,}000}{72{,}799.07}\right)^{1/10} - 1$$

$$= (1.37364393)^{.1} - 1$$

$$= 1.032256 - 1$$

$$= .032256 \approx 3.23\%$$

If rate of return to maturity = 3.23%,
what is the meaning of the 3.20% yield to maturity?

Coupon Bond Prices

Maturities: 5, 10 and 30 years

$10,000 annual coupons, $1,000,000 par value, 4% interest rate

$$\text{5 year:} \quad P = \frac{10{,}000}{1.04^1} + \frac{10{,}000}{1.04^2} + \cdots + \frac{1{,}010{,}000}{1.04^5} = 866{,}445.33$$

$$\text{10 year:} \quad P = \frac{10{,}000}{1.04^1} + \frac{10{,}000}{1.04^2} + \cdots + \frac{1{,}010{,}000}{1.04^{10}} = 756{,}673.13$$

$$\text{30 year:} \quad P = \frac{10{,}000}{1.04^1} + \frac{10{,}000}{1.04^2} + \cdots + \frac{1{,}010{,}000}{1.04^{30}} = 481{,}233.00$$

Perpetuity – Definition

- A *perpetuity* is a set sum of cash received forever
- Ordinary preferred stock can often be treated as a perpetuity because it pays a constant dividend and never matures
- In Corporate Finance, why may one use perpetuity analysis?

PV of a Perpetuity

PV of a Perpetuity with the first payment starting one year from today:

$$PV = CF / k$$

Where:

PV = Present value

CF = Cash flow of perpetuity amount

k = Interest rate (expressed as a decimal)

PV of a Perpetuity – Example

If a preferred stock pays a constant dividend of $25 per year and there is a required rate of return of 12.5%, what is the stock worth?

$$PV = CF / k$$

Growing Perpetuities

PV of a Perpetuity with the first payment starting one year from today, growing at a constant rate forever:

$$PV = CF_1 / (k - g)$$

Where:

PV = Present value

CF_1 = Cash flow of perpetuity amount

k = Interest rate (expressed as a decimal)

g = Growth rate (a constant) expressed as a decimal

Weighted Averages

- Method for adjusting measured outcomes when factors impacting outcome don't deserve equal weighting
  - Unequally weighted portfolios
  - Future states (outcome) that are not equally likely
- Weights reflect the proportion or probability of input
- Weights must total to 100%

$$\overline{X}_w = \sum_{i=1}^{N} w_i X_i$$

Weighted Mean Example

- Simple average (sum of observed values/# observations) accurate for equally weighted portfolios or equally probable future states
- Confirm that the Arithmetic Mean is 20%

Investment Type	Portfolio Allocation	Annual Return
Stocks	33%	10%
Bonds	33%	20%
Property	33%	30%

Weighted Mean Example

- Now, let’s change the asset allocation weighting in the portfolio

- We get the weighted mean by multiplying portfolio allocation by the annual return for each asset

- Then total the weighted results for all 3 assets
  - Unlike arithmetic mean, we do not divide by the number of observations/categories

Investment Type	Portfolio Allocation	Annual Return	Weighted Result
Stocks	50%	10%	5.0%
Bonds	30%	20%	6.0%
Property	20%	30%	6.0%
Total	100%		17.0%

Probabilities

- Corporate finance analyses will often use probability-weighted future states to reflect uncertainties
- Simulations of many variables, using Monte Carlo methods based on statistical assumptions will generate value estimates that reflect many possible futures
- Input variables (e.g. interest rate volatility, real GDP growth rates, mortgage prepayment rates) can be altered to establish a range of likely estimates
- Use of more sophisticated techniques is no guarantee of accuracy

Dealing with Complex Problems

You may run into more complex problems using annuities. For example:

- What investment today will grow to provide a specified annuity at a specified date in the future?
- What regular investment will grow to provide a specified annuity in the future (e.g. retirement planning)?

As with many real world problems, solution necessitates a multi-stage solution

Complex Problems – Example

If you have $25,000 and want to have $1,000,000 in 30 years, how much do you have to invest each year at 9% to achieve this goal?

Complex Problems – Solution

- Solve in two stages
  - What is the value of 25,000 invested over 30 years at 9%?
  - What sum is required to make up the balance?

Complex Problems – Solution Stage One

- Stage One:

$$PV = 25{,}000 \times (1 + 0.09)^{30}$$

$$PV = 25{,}000 \times (1.09)^{30}$$

$$PV = 25{,}000 \times 13.268$$

$$PV = 331{,}700$$

Complex Problems – Solution Stage Two

- Stage Two

$1{,}000{,}000 - 331{,}700 = 668{,}300$

$$668{,}300 = CF\,[((1 + i)^N - 1) / i]$$

$$= CF\,[(1.09)^{30} - 1 / 0.09]$$

$$= CF\,[(13.268 - 1)/0.09]$$

$$= CF \times 136.311$$

$668{,}300 / 136.311 = CF$

CF = 4,902.94

Exercise Two

Working individually, complete exercise on present and future value of annuities and perpetuities

Additional Class Exercise

- Royalty Trusts are investment vehicles which are involved in natural resource extraction and distribution. The trust does not pay corporate income taxes as long as it distributes nearly all its income.
  - If oil is $100 a barrel and a company pumps 1,000,000 barrels a year for 10 years, should it accept a bid of $400,000,000 if it requires a 20% return?
- Aside from a simple determination of whether or not the bid would result in a return exceeding the hurdle rate, what other factors must the owners of the Royalty Trust weigh when assessing the attractiveness of the bid?

Additional Class Exercise

- Since the future price of a commodity is unpredictable it is often necessary to preform an analysis that incorporates a range of possible outcomes. What if oil has a 20% chance of being $80; a 50% chance of being $100; and, a 30% chance of being $120?.
- Calculating a probability weighted outcome is a means of projecting a return that reflects some of the uncertainty related to future results
- In most real world settings, formulating realistic assumptions is usually much more difficult than performing the calculations

Module 3

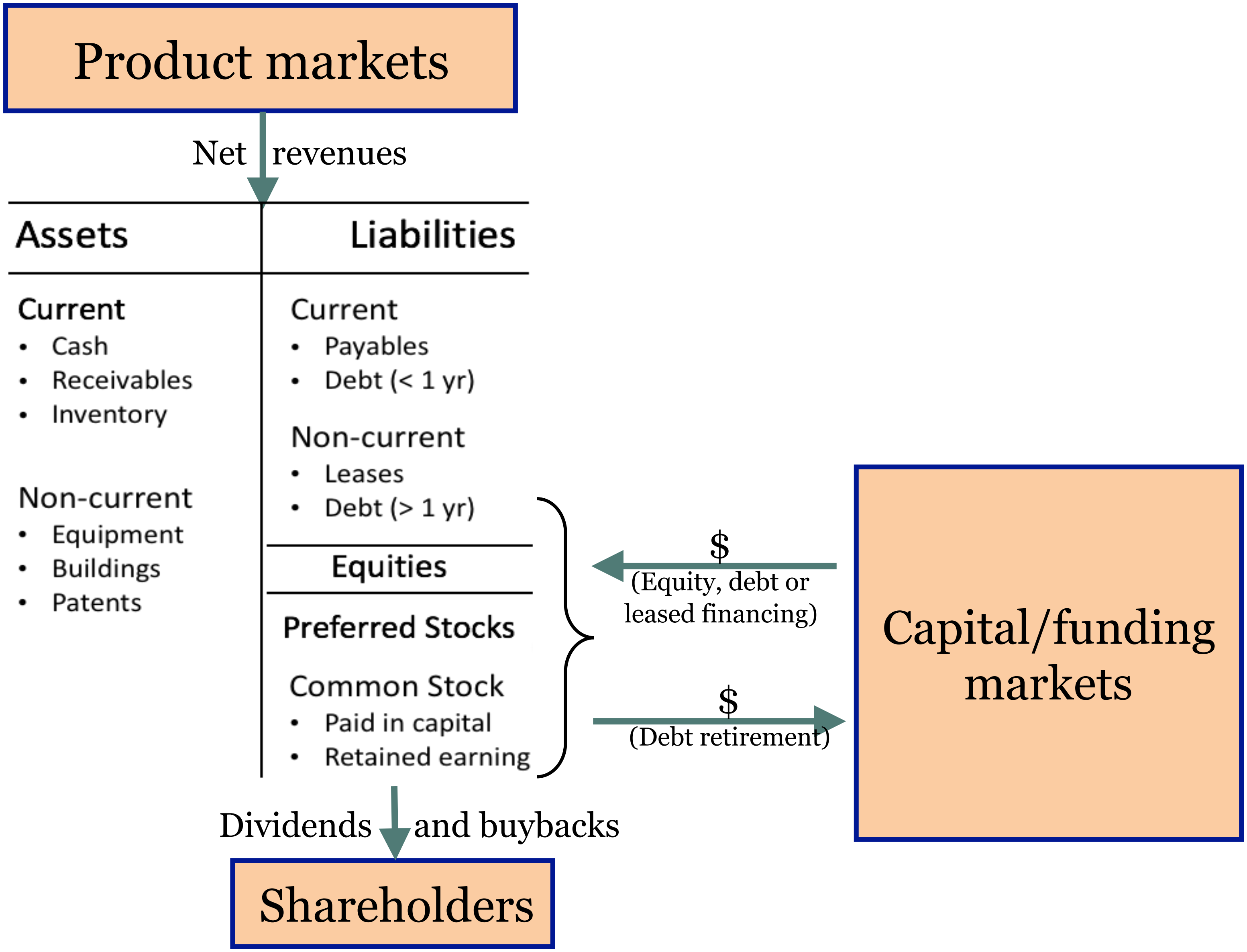
Capital Budgeting: Investment Project Selection Methodologies

Capital Budgeting

- Capital budgeting is the process of selecting new long-term asset investments

- The goal of capital budgeting is to maximize the market value of the firm’s existing equity shares

Corporate Finance Decisions



Product markets

Net revenues

Assets

Current
- Cash
- Receivables
- Inventory

Non-current
- Equipment
- Buildings
- Patents

Liabilities

Current
- Payables
- Debt (< 1 yr)

Non-current
- Leases
- Debt (> 1 yr)

Equities

Preferred Stocks

Common Stock
- Paid in capital
- Retained earning

$
(Equity, debt or leased financing)

$
(Debt retirement)

Capital/funding markets

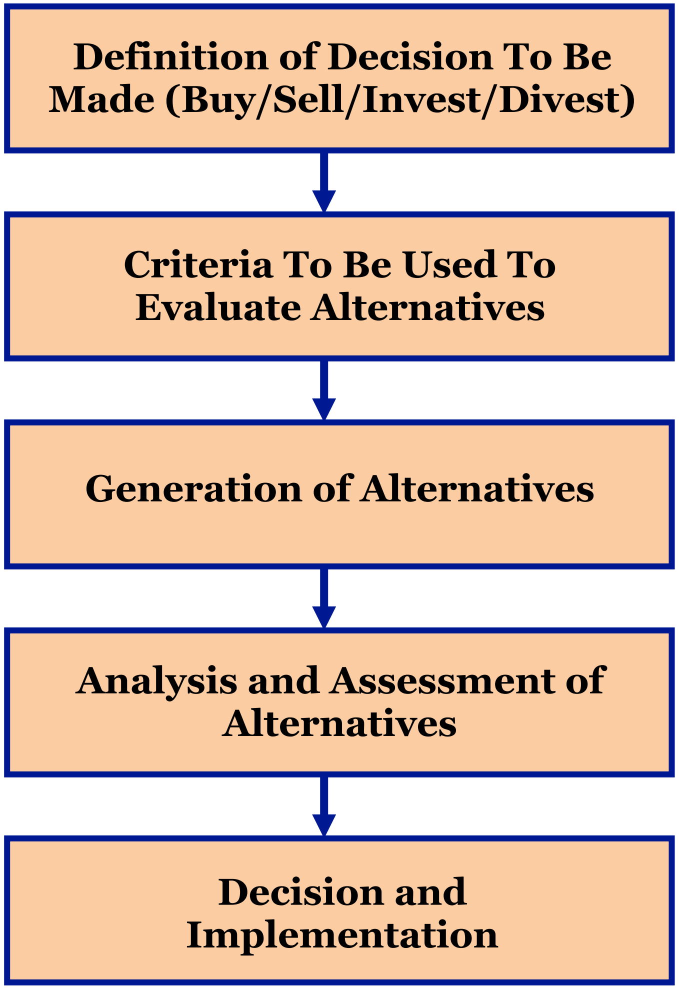
Dividends and buybacks

Shareholders

Capital Budgeting Rule

- Invest only in projects that earn a return greater than the ‘Opportunity Cost of Capital’
- Opportunity cost of capital is what a shareholder could earn on an investment of similar risk

Decision Making Framework



Definition of Decision To Be Made (Buy/Sell/Invest/Divest)

Criteria To Be Used To Evaluate Alternatives

Generation of Alternatives

Analysis and Assessment of Alternatives

Decision and Implementation

Project Evaluation

- All evaluation starts with the initial cost (investment) and then looks at future benefits (cash flows)

- There are three main approaches:
  - Payback or Payback Period (PBP)
  - Internal Rate of Return (IRR)
  - Net Present Value (NPV)

- There are also variations of PBP and IRR
  - Discounted Payback Period (DPBP)
  - Modified Internal Rate of Return (MIRR)

Identifying Cash Flows

- It is important to include only costs and cash flows that are relevant to the project

- We will consider these issues:
  - Incremental costs / cash flows
  - Sunk costs
  - Overhead costs
  - Opportunity costs
  - After-tax
  - Nominal vs. real cash flows

Identifying Cash Flows

Incremental costs / cash flows

- If introduction of a new project causes sales and cash flows of an existing project to fall, this must be included
- Sales increase will typically necessitate additional working capital (e.g. inventories, accounts receivable, etc.) to support the higher level of business activity
- Investment in plant or equipment will often entail cash flows, positive (e.g. sale or salvage value) or negative (e.g. clean-up or disposal costs) which must be accounted for

Identifying Cash Flows

Sunk Costs

- Capital budgeting decisions take place ‘today’
- Only future cash inflows and outflows should impact investment decision
- Previous expenditures should not influence investment decisions being taken today
  - Costs of evaluating projects
  - Prior net investment/operating expenditures

Identifying Cash Flows

Overhead costs

- Overhead costs (e.g., internal ‘rent’ of floor space) are often allocated for financial reporting purposes
- Only costs that are directly relevant to the project should be included

Opportunity costs

- Optimal allocation of funds for investment purposes decision needs to account of other potential uses of cash (e.g. alternative fixed investment in property, plant or equipment)

Identifying Cash Flows

After-tax

- The discount rate used to evaluate projects is based on a tax-adjusted cost of capital

- Consequently, returns should also be evaluated on an after-tax basis

Identifying Cash Flows

Nominal or Real

- Generally use nominal cash flows (i.e., allowing for future inflation)
- In high inflation environments, may use real cash flows (eliminating the impact of future inflation)
- The key point is to match nominal cash flows with a nominal discount rate and real cash flows with a real discount rate

Exercise Three

In groups, discuss the exercise on identifying relevant costs

Project Analysis Conventions

- Cash outlay at beginning of project occurs at Time ‘0’ and is not discounted
- Cash inflows are assumed to occur at the end of each year
- For discounting purposes, the beginning of one year is no different from the end of the previous year

Payback Period

The period that it takes the project to recoup the project cost (CF_0)

Decision Rule:

When deciding between two or more projects, choose the one with the shortest payback period.

Payback - Example

Project	CF_0	CF_1	CF_2	CF_3
A	(2000)	1000	1000	0
B	(2000)	500	1000	5000

Which project should you choose?

Payback Period

Advantages

- ✓Easy to use
- ✓Commonly used as short-hand among operating managers
- ✓Takes some account of risk
- ✓Good as a first round screening method

Disadvantages

- ✓No account of the time value of money
- ✓Does not take account of risk and uncertainty of cash flows after the payback period

Discounted Payback Period

The period that it takes the project's discounted cash flows to recoup the project cost (CF_0)

Decision Rule:

When deciding between two or more projects, choose the one with the shortest discounted payback period.

Discounted Payback - Example

Project	CF_0	CF_1	CF_2	CF_3
A	(2000)	1000	1000	0
(present value)	(2000)	909	826	0
B	(2000)	500	1000	5000
(present value)	(2000)	454	826	3757

Which project should you choose?

Discounted Payback

Advantages

- ✓Quick to use
- ✓Takes account of time value of money
- ✓Easily understood

Disadvantages

- ✓No consideration of cash flows after payback period
- ✓Short-term viewpoint

Internal Rate of Return (IRR) - Definition

- IRR is the average annual rate of return earned over a project's life assuming
  - Net cash flows are reinvested at that rate
  - Future investment outlays can be financed at that same rate
- Quantifies the amount by which a project's net cash inflows exceed outflows on an annualised percentage basis (adjusted for time value of money considerations)
- The discount rate at which a project had a NPV of zero

Computing Internal Rate of Return

- Inputs
  - Initial investment outlay
  - Amount and timing of cash flows
  - Time horizon (life of project)
  - Cash flows include any salvage value/sales proceeds at end of investment horizon

- Assumptions
  - All future periods will have net positive cash flows
  - Future cash flows can be reinvested at IRR

Determining the Internal Rate of Return (IRR)

- Determine necessary cash flows for calculation
  - Initial investment (at time 0)
  - Project periodic cash inflows (including terminal/salvage value)
  - Same inputs would be used in NPV calculation
- IRR is the discount rate (k) that results in NPV = 0
- There is no closed form solution for calculating IRR
- Solved via iterative process
  - Trial and error process
  - Assume a discount rate and calculate present value of cash flow
  - Repeat until finding a discount rate resulting in PV so calculated is equal to initial investment

Internal Rate of Return (IRR) – How to do it

$$CF_0 = \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_N}{(1+IRR)^N}$$

or

$$CF_0 - \frac{CF_1}{(1+IRR)^1} + \frac{CF_2}{(1+IRR)^2} + \ldots + \frac{CF_N}{(1+IRR)^N} = 0$$

Where:

CF_N = cash flow at time N

IRR = internal rate of return

Internal Rate of Return (IRR) - Example

Metroset Production is considering building a new factory…

- Investment Cost: $100,000 (assumed at time ‘0’)
- Cash Flows:
  - Year 1 - $35,000
  - Year 2 - $55,000
  - Year 3 - $75,000

What is the IRR? Should they build the factory?

Internal Rate of Return– Solution (Excel)

	A	B	C	D	E
1					
2		-100,000			
3		35000			
4		55000			
5		75000			
6	=IRR(B2;B5)	26%			
7					

Problematic Aspects IRR

IRR does has some significant problems:

- Assumption of cash flows being re-invested at IRR rate
- No explicit adjustment for risk of project
- There can be multiple solutions (i.e. multiple IRRs) when projects have future periods with negative cash flows
  - Net operating losses
  - Future investment outlays
  - Remediation/wind-down costs

Modified IRR (MIRR)

- The discount rate that equates the PV of cash outflows for a given capital project with the PV of the ‘terminal value’ of cash inflows from the project
- Assumes that cash inflows are reinvested at the project’s cost of capital
    - A better (or at least more conservative) assumption than reinvestment at the IRR

MIRR

$$\sum_{T=1}^{N} \frac{COF_T}{(1+k)^T} = \frac{\sum_{T=1}^{N} CIF_T (1+k)^{N-T}}{(1+MIRR)^N}$$

Where:

MIRR = modified internal rate of return

COF = cash outflows

CIF = cash inflows

k = cost of capital

MIRR - Example

Metroset Production is considering building a new factory…

- Investment Cost: $100,000 (assumed at time ‘0’)
- Opportunity Cost of Capital: 9.38%
- Cash Flows:
  - Year 1 - $35,000
  - Year 2 - $55,000
  - Year 3 - $75,000

What is the MIRR?

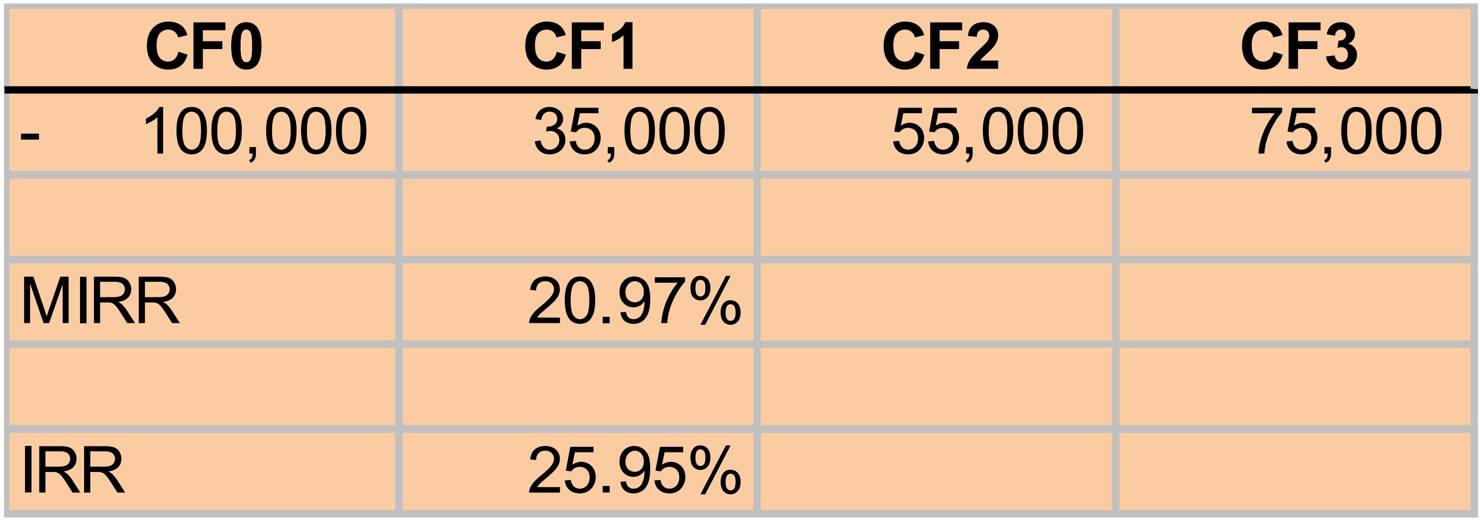
MIRR - Solution

- Investment Cost: \$100,000 (assumed at time ‘0’)
- Opportunity Cost of Capital: 9.38%

Cash Flow	Future Value at Year 3	Multiplication Factor
Year 1: \$35,000	**\$41,874**	$(1+0.0938)^2$
Year 2: \$55,000	**\$60,159**	$(1+0.0938)^1$
Year 3: \$75,000	**\$75,000**	**1**
	\$177,033	
$(\$177{,}033 / \$100{,}00)^{1/3} - 1 = 0.2097 = 20.97\%$		

Modified IRR - Solution

This shows the Excel spreadsheet solution:



CF0	CF1	CF2	CF3
- 100,000	35,000	55,000	75,000
MIRR	20.97%		
IRR	25.95%		

Note that the IRR is significantly higher than MIRR. This is because it assumes cash is reinvested at 26%, not 9.38%.

Net Present Value (NPV)

- NPV discounts all future cash flows by the project's cost of capital (k_A) and then sums up these numbers
- The initial investment is subtracted from this sum to determine the NPV of the investment

Decision Rule:

If NPV > 0, accept the investment

Discount Rate Used in NPV Analysis

- The appropriate discount rate is a risk-adjusted opportunity cost of the capital
- The opportunity cost of capital is the return that could be earned on investments of similar risk
- Companies use different discount rates across projects
  - Intel corporation uses different hurdle rates for semiconductor versus software investments
  - Energy companies use different discount rates for drilling projects versus pipelines
- In some companies it is called the “hurdle rate”

Methodologies for determining appropriate discount rates will be discussed on Day Two

Net Present Value / Discounted Cash Flow

Project	CF_0	CF1	CF2	CF3
A	(2000)	1000	1000	0
(present value)	*(2000)*	*909*	*826*	*0*
B	(2000)	500	1000	5000
(present value)	*(2000)*	*454*	*826*	*3757*

Which project should you choose (if any)?

Advantages of NPV

✓Allows for timing of cash flow

✓All cash flows considered

✓Clear decisions

✓Risk easily incorporated via the discount rate

Net Present Value (NPV) - Formula

$$NPV = \sum_{T=0}^{N} \frac{CF_T}{(1+k)^T}$$

or

$$NPV = -CF_0 + \frac{CF_1}{(1+k)^1} + \frac{CF_2}{(1+k)^2} + \ldots + \frac{CF_N}{(1+k)^N}$$

Where:

NPV = Net Present Value

T_o to T_N = Time horizon

CF = Cash flows (CF_o is the initial investment)

k = Discount rate or WACC

Note: $(1+k)^T$ is also known as the Present Value Factor or the Discount Factor

Net Present Value (NPV) - Example

Metroset Production is considering building a new factory…

- Investment Cost: $100,000 (assumed at time ‘0’)
- Opportunity Cost of Capital: 9.38%
- Cash Flows:
  - Year 1 - $35,000
  - Year 2 - $55,000
  - Year 3 - $75,000

What is the NPV? Should they build the factory?

Net Present Value (NPV) - Solution

$$NPV = \sum_{t=0}^{n} \frac{CF_1}{(1+k)^t}$$

$$= -100{,}000 + \frac{35{,}000}{(1.0938)^1} + \frac{55{,}000}{(1.0938)^2} + \frac{75{,}000}{(1.0938)^3}$$

$$= -100{,}000 + 31{,}999 + 45{,}971 + 57{,}312$$

$$= 35{,}282$$

The new factory has a positive NPV and therefore should be accepted

Net Present Value (NPV) – Solution (Excel)

	A	B	C	D	E
1					
2		9%	Annual discount rate		
3		-100,000	Initial investment		
4		35000	First year cash flow		
5		55000	Second year cash flow		
6		75000	Third year cash flow		
7	=NPV(B2,B4:B6)+B3	$35,282.10	Net present value		

Profitability Index

- The profitability index tries to evaluate projects of different scale

$$\text{Profitability Index} = \text{NPV} / \text{CF}_0$$

Exercise Four

Individually, complete the exercise on NPV and IRR

NPV vs. IRR

IRR's advantages

✓Easier to understand by some - % measure

✓Can be calculated without specifying a required return

NPV's advantages

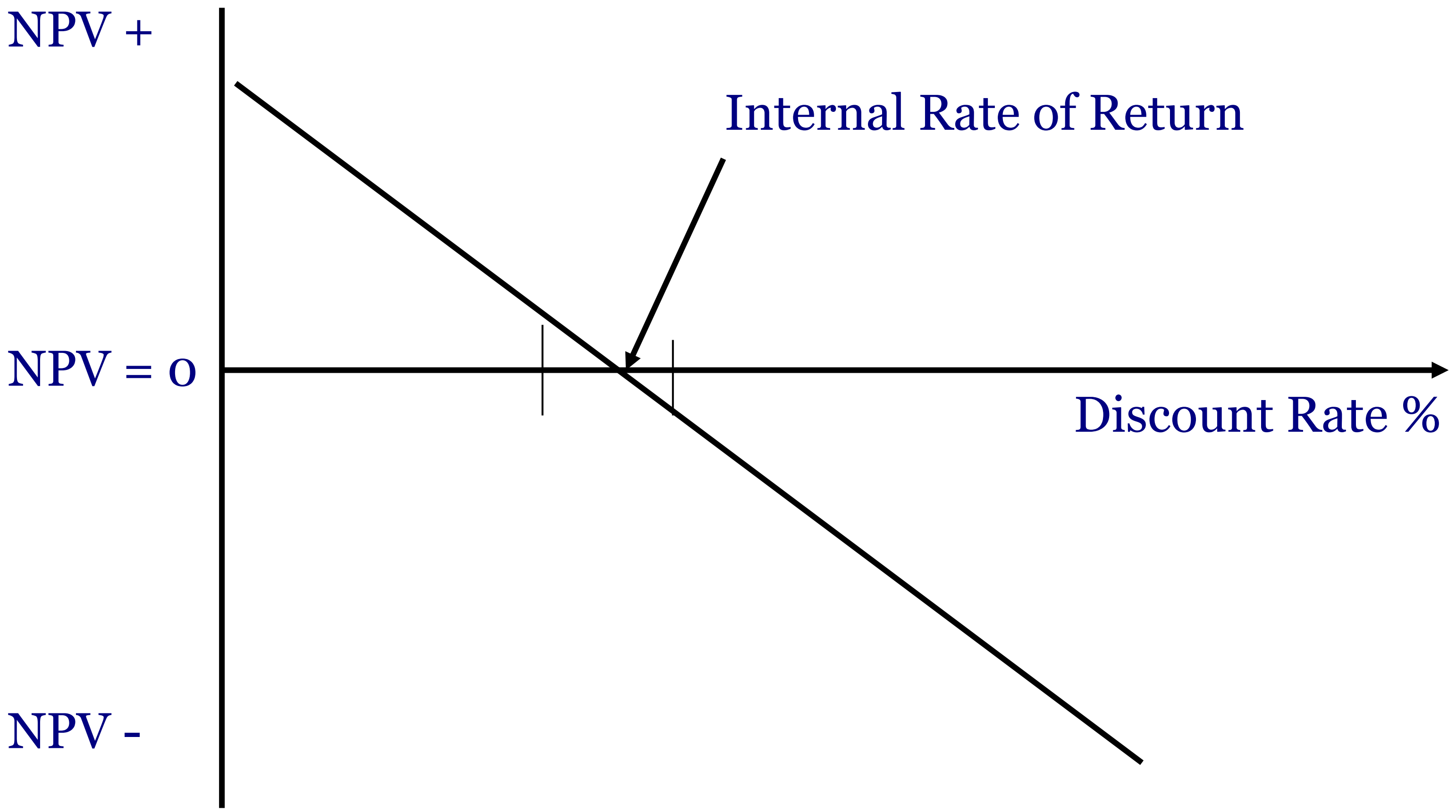
✓No multiple answers

✓Correct ranking of mutually exclusive projects

✓Shows monetary (absolute) change in wealth

✓Additive

Relationship Between IRR and NPV



Review of Day One Learning Objectives

- Differentiate between simple and compound interest
- Solve simple and compound interest problems
- Solve problems using time value of money concepts
- Differentiate between nominal and effective interest rates
- Evaluate capital projects using payback period and discounted payback
- Differentiate between present value and future value
- Calculate IRRs for different projects